Total correlations of the diagonal ensemble herald the many-body localization transition

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The intriguing phenomenon of many-body localization (MBL) has attracted significant interest recently, but a complete characterization is still lacking. We introduce the total correlations, a concept from quantum information theory capturing multi-partite correlations, to the study of this phenomenon. We demonstrate that the total correlations of the diagonal ensemble provides a meaningful diagnostic tool to pin-down, probe, and better understand the MBL transition and ergodicity breaking in quantum systems. In particular we show that the total correlations has sub-linear dependence on the system size in delocalized, ergodic phases, whereas we find that it scales extensively in the localized phase developing a pronounced peak at the transition.

Reference: arXiv:1504.06872.

The total correlations

- Definition: Consider an N-partite system, then

$$T(\rho) := \sum_{m=1}^{N} S(\rho_m) - S(\rho).$$

Where $S(\rho) := -\mathrm{tr}(\rho \log_2 \rho)$ is the von Neumann entropy.

- Intuition: Measures distinguishability from product state:

$$T(\rho) = \min_{\pi \text{product state}} S(\rho \| \pi)$$

Where $S(\rho\|\sigma) := -\mathrm{tr}(\rho \log_2 \sigma) - S(\rho) \geq \|\rho - \sigma\|_1^2/2$ is the relative entropy.

Ergodicity

- Starting point:
Ergodic Hypothesis: ergodic $\Leftrightarrow$ systems explore phase space uniformly
$\Rightarrow$ infinite time average $=$ microcanonical average
Essentially impossible in QM hence ask for less:
- Definition (in words):

> A pair of Hamiltonian and initial state is ergodic (as oppose to MBL) if it explores at least a constant fraction of the available Hilbert space.

- Quantify: For a fixed initial state ρ and non-degenerate Hamiltonian H define the dephased or time-averaged state (diagonal ensemble)

$$\omega := \sum_n |E_n\rangle\langle E_n| \, \rho \, |E_n\rangle\langle E_n| = \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt \, \mathrm{e}^{-itH} \rho \, \mathrm{e}^{itH},$$

with $|E_n\rangle$ eigenvectors of H.
A family of systems of increasing size N is ergodic if $\exists \lambda > 0$ such that for most product initial states from some subspace (i.e., fixed magnetization) of dimension d it holds (with high probability over the disorder average)

$$S(\omega) \geq \log_2(\lambda \, d)$$

Scaling of T with N

- Non-ergodic: As $T(\omega)$ involves the sum $\sum_{m=1}^{N} S(\omega_m)$ of the N subsystem entropies:

$$T(\omega) \propto N$$

- Ergodic: If a family of disordered systems is ergodic however, then $\exists \lambda > 0$ such that for most product initial states with high probability

$$T(\omega) \leq \sum_{m=1}^{N} S(\omega_m) - \log_2(\lambda \, d).$$

For a spin-1/2 chain the magentization zero subspace has $d = \binom{N}{N/2} = N!/\left(\frac{N}{2}!\right)^2 \geq \sqrt{8\pi}\,\mathrm{e}^{-2} 2^N/\sqrt{N}$ and $S(\omega_m) \leq \log_2 2 = 1$, so that

$$T(\omega) \leq \log_2(N)/2 - \log_2(\lambda \sqrt{8\pi}\,\mathrm{e}^{-2}).$$

- Intuition: Transport in ergodic systems makes parts so mixed that their distinguishability from the closest product state only grows logarithmically.

Heisenberg spin-1/2 chain ...

... of N sites with coupling constants J, J_{zz} and disorder strenght h:

$$H = \sum_{i=1}^{N} \left[J \left(\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1}\right) + J_{zz} \, \sigma_z^i \sigma_z^{i+1} + h_i \, \sigma_z^i \right]$$

Where we set $J = J_{zz} = 1$ and take $h_i \in [-h, h]$ uniformly distributed.
Then it has a MBL transition at $h = h_c \in [2, 4]$ [1].

Numerical results

1. Restrict to zero magnetization subspace
2. For product initial states compupte $T(\omega)$.
3. Average over all initial states and disorder $\rightarrow \overline{T}(\omega)$.

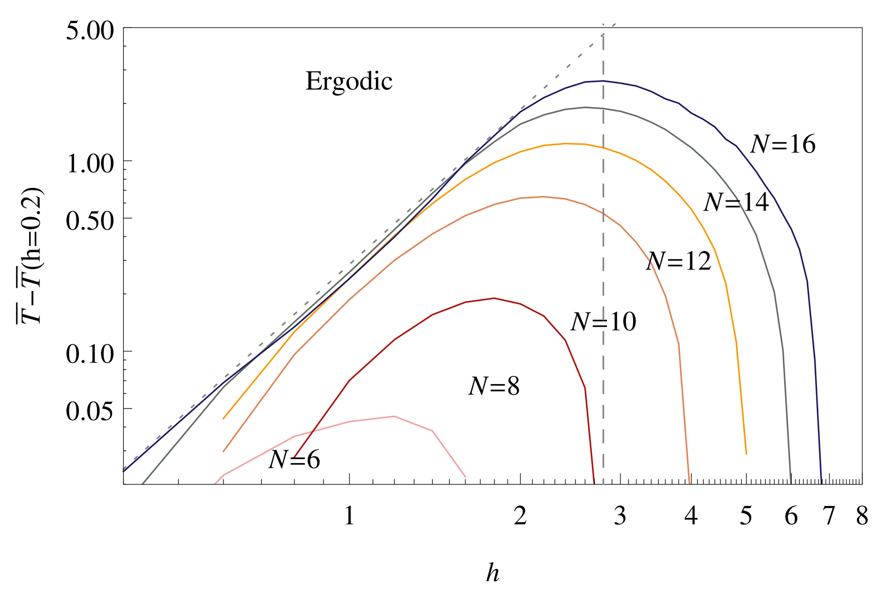


Figure 1: Initial increase for small h is power-law like with exponent 2.7(2). The deviation maks beginning of the MBL transition region.

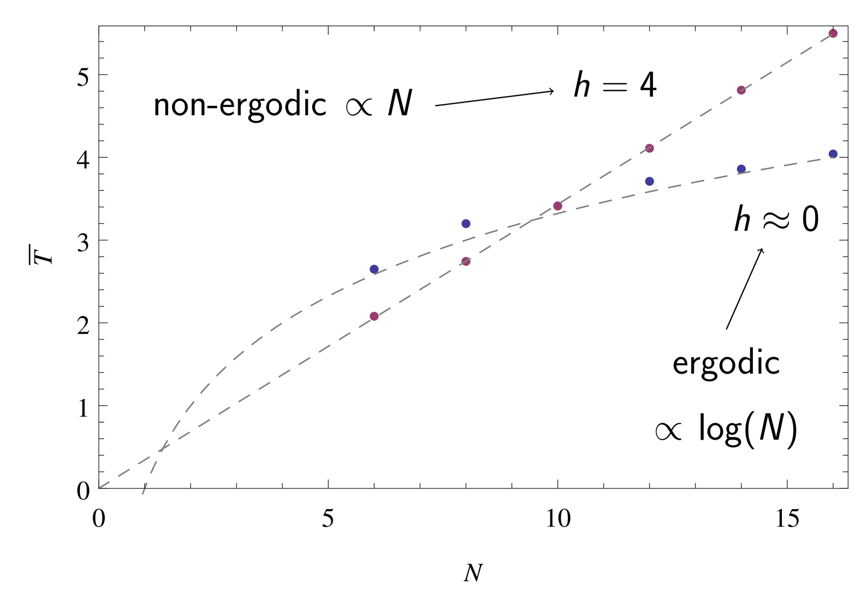


Figure 3: For small h we see logarithmic growth, as expected in an ergodic system, and at large h linear growth.

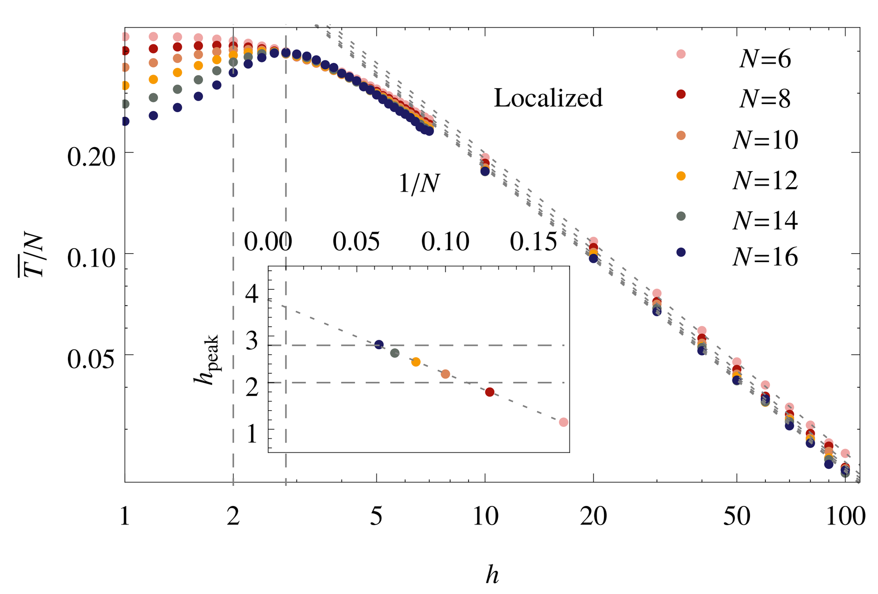


Figure 2: For high h, $\overline{T}$ decays as a power-law with exponents of $-0.9(1)$. Inset shows position of peaks in Figure 1.

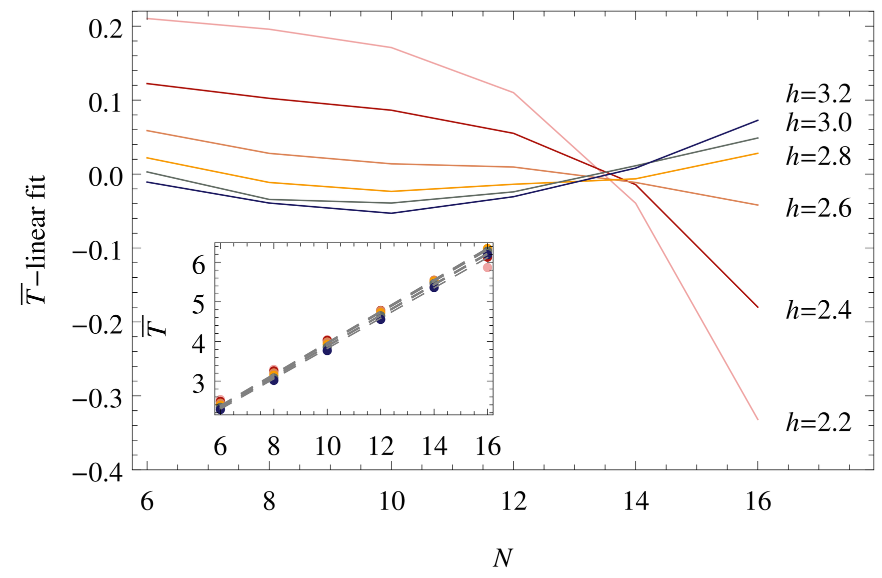
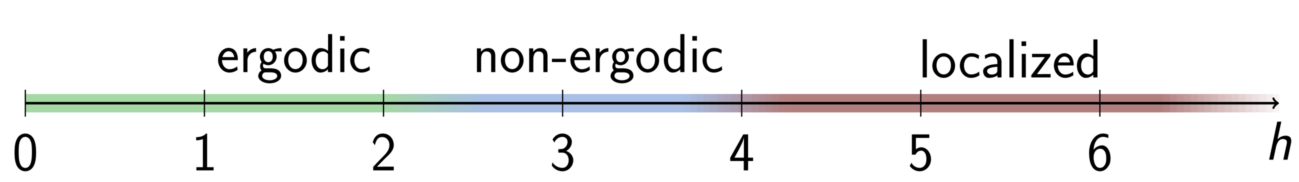


Figure 4: Difference between $\overline{T}$ and best possible linear fit for different values of h. Crossover from a nearly linear scaling to a sub-linear scaling happens at $h = 2.6(2)$ (robust against omitting data points and holds also for affine fits). The inset shows raw data.

Conclusions

- Total correlations in the diagonal ensemble signal the MBL transition
- They expose how this transition involves reorganization of correlations
- Results suggests two step transition in line with [2, 3, 4]



[1] A. Pal and D. A. Huse, Phys. Rev. B **82**, 174411 (2010).
[2] A. De Luca, B. L. Altshuler, V. E. Kravtsov, and A. Scardicchio, Phys. Rev. Lett. **113**, 046806 (2014).
[3] B. L. Altshuler, Y. Gefen, A. Kamenev, and L. S. Levitov, Phys. Rev. Lett. **78**, 2803 (1997).
[4] M. Serbyn and J. E. Moore, arXiv:1508.07293 (2015).