Total correlations of the diagonal ensemble herald the many-body localization transition

J. Goold, C. Gogolin, S. R. Clark, J. Eisert, A. Scardicchio, and A. Silva

The intriguing phenomenon of many-body localization (MBL) has attracted significant interest recently, but a complete characterization is still lacking. We introduce the total correlations, a concept from quantum information theory capturing multi-partite correlations, to the study of this phenomenon. We demonstrate that the total correlations of the diagonal ensemble provides a meaningful diagnostic tool to pin-down, probe, and better understand the MBL transition and ergodicity breaking in quantum systems. In particular we show that the total correlations has sub-linear dependence on the system size in delocalized, ergodic phases, whereas we find that it scales extensively in the localized phase developing a pronounced peak at the transition.


The total correlations

Definition: Consider an N-partite system, then

\[ T(\rho) := \sum_{n=1}^{N} S(\rho_{n}) - S(\rho). \]

Where \( S(\rho) := -\text{tr}(\rho \log_2 \rho) \) is the von Neumann entropy.

Intuition: Measures distinguishability from product state:

\[ T(\rho) = \min \{ S(\rho || \pi) \} \]

Where \( S(\rho || \pi) := -\text{tr}(\rho \log_2 \pi) - S(\rho) \geq \|\rho - \|\pi\|/2 \) is the relative entropy.

Ergodicity

Starting point:

Ergodic Hypothesis: ergodic \( \iff \) systems explore phase space uniformly \( \Rightarrow \) infinite time average = microcanonical average

Essentially impossible in QM hence ask for less:

Definition (in words):

A pair of Hamiltonian and initial state is ergodic (as oppose to MBL) if it explores at least a constant fraction of the available Hilbert space.

Quantity: For a fixed initial state \( \rho \) and non-degenerate Hamiltonian \( H \) define the dephased or time-averaged state (diagonal ensemble)

\[ \omega := \sum_n \{ E_n\}_\rho \{ E_n\}_\rho = \frac{1}{T} \int dt e^{-itH} \rho e^{itH}, \]

with \( \{ E_n\} \) eigenvectors of \( H \).

A family of systems of increasing size \( N \) is ergodic if \( \exists \lambda > 0 \) such that for most product initial states from some subspace (i.e., fixed magnetization) of dimension \( d \) holds (with high probability over the disorder average)

\[ S(\omega) \geq \log_2(\lambda d) \]

Scaling of \( T \) with \( N \)

Non-ergodic: As \( T(\omega) \) involves the sum \( \sum_{n=1}^{N} S(\omega_{n}) \) of the \( N \) subsystem entropies:

\[ T(\omega) \propto N \]

Ergodic: If a family of disordered systems is ergodic however, then \( \exists \lambda > 0 \) such that for most product initial states with high probability

\[ T(\omega) \leq \sum_{n=1}^{N} S(\omega_{n}) - \log_2(\lambda d). \]

Intuition: Transport in ergodic systems makes parts so mixed that their distinguishability from the closest product state only grows logarithmically.

Heisenberg spin-1/2 chain . . .

. . . of \( N \) sites with coupling constants \( J, J_{zz} \) and disorder strenght \( h \):

\[ H = \sum_{i=1}^{N} \left[ J_{i} \sigma_{i}^{+} \sigma_{i}^{-} + J_{i}^{zz} \sigma_{i}^{+} \sigma_{i}^{-} + h \sigma_{i}^{z} \right] \]

Where we set \( J = J_{zz} = 1 \) and take \( h \in [-h, h] \) uniformly distributed.

Then it has a MBL transition at \( h = h_{\text{c}} \in [2, 4] \) [1].

Numerical results

Restrict to zero magnetization subspace

For product initial states compute \( T(\omega) \).

Average over all initial states and disorder \( \rightarrow T(\omega) \).

Results suggest two step transition in line with [2, 3, 4]

Conclusions

Total correlations in the diagonal ensemble signal the MBL transition

They expose how this transition involves reorganization of correlations

Results suggests two step transition in line with [2, 3, 4]

