Locality of temperature
structural properties of thermal states

Christian Gogolin

ICFO - The Institute of Photonic Sciences

MPQ - Max Planck Institute of Quantum Optics

New trends in complex quantum systems dynamics
2015-05-27

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structural properties of thermal states

Not as technical as it sounds!

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Motivation I: Measuring temperature

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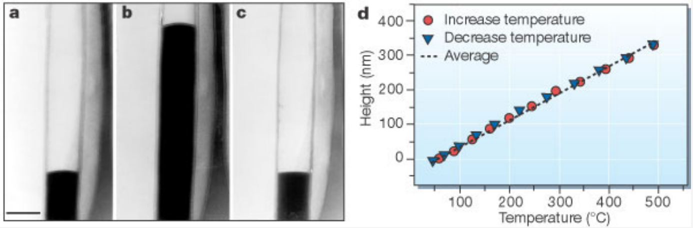

[1] Y. Gao and Y. Bando, Nature, 415.6872 (2002), 599

Motivation II: Coherent dynamics of large systems

Science 337, 1318 (2012):

Relaxation and Prethermalization in an Isolated Quantum System

M. Gring,[1] M. Kuhnert,[1] T. Langen,[1] T. Kitagawa,[2] B. Rauer,[1] M. Schreitl,[1] I. Mazets,[1,3] D. Adu Smith,[1] E. Demler,[2] J. Schmiedmayer[1,4*]

Understanding relaxation processes is an important unsolved problem in many areas of physics. A key challenge is the scarcity of experimental tools for the characterization of complex transient states. We used measurements of full quantum mechanical probability distributions of matter-wave interference to study the relaxation dynamics of a coherently split one-dimensional Bose gas and obtained comprehensive information about the dynamical states of the system. After an initial rapid evolution, the full distributions reveal the approach toward a thermal-like steady state characterized by an effective temperature that is independent from the initial equilibrium temperature of the system before the splitting process. We conjecture that this state can be described through a generalized Gibbs ensemble and associate it with prethermalization.

Despite its fundamental importance, a general understanding of how isolated quantum many-body systems approach thermal equilibrium is still elusive. Theoretical concepts such as the quantum ergodic theory or the eigenstate thermalization hypothesis (1–3) infer requirements for a system to be able to undergo relaxation, but it is still unclear on what time scale this occurs. In situations in which conservation laws inhibit efficient relaxation, many-body systems are expected to display a complex behavior. An intriguing phenomenon that has been suggested in this context is prethermalization (4), a general concept that is predicted to be applicable to a large variety of physical systems (5–9). In the present understanding, prethermalization is characterized by the rapid establishment of a quasi-stationary state that already exhibits some equilibrium-like properties. Full relaxation to the

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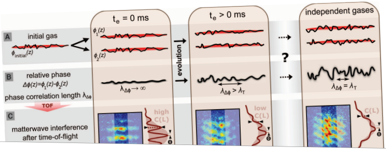
*To whom correspondence should be addressed. E-mail: schmiedmayer@atomchip.org



LETTERS
PUBLISHED ONLINE: 8 SEPTEMBER 2013 | DOI: 10.1038/NPHYS2739
nature physics

Local emergence of thermal correlations in an isolated quantum many-body system

T. Langen*, R. Geiger, M. Kuhnert, B. Rauer and J. Schmiedmayer*

Understanding the dynamics of isolated quantum many-body systems is a central open problem at the intersection between statistical physics and quantum physics. Despite important theoretical effort[1], no generic framework exists yet to understand when and how an isolated quantum system relaxes to a steady state. Regarding the question of how, it has been conjectured[2,3] that equilibration must occur on a local scale in systems where correlations between distant points can establish only at a finite speed. Here, we provide the first experimental observation of this local equilibration hypothesis. In our experiment, we quench a one-dimensional Bose gas by coherently splitting it into two parts. By monitoring the phase coherence between the two parts we observe that the thermal correlations of a prethermalized state[4,5] emerge locally in their final form and propagate through the system in a light-cone-like evolution. Our results underline the close link between the propagation of correlations[2,3,6,7] and relaxation processes in quantum many-body systems.

It has been theoretically suggested that relaxation in generic isolated quantum many-body systems proceeds through the dephasing of the quantum states populated at the onset of the non-equilibrium evolution[8,9]. It is generally believed that this dynamically leads to relaxed states that can be well described either by the usual thermodynamical ensembles or by generalized Gibbs ensembles that take into account dynamical constraints[10]. However, it remains an open question how these relaxed states form dynamically, and in particular, whether they emerge gradually on a global scale, or appear locally and then spread in space and time[3].

Ultracold atomic gases offer an ideal test bed to explore such quantum dynamics. Their almost perfect isolation from the environment and the many available methods to probe their quantum states make it possible to reveal the dynamical evolution of a many-body system at a very detailed level[4,7,11–16]

well, thereby realizing the matter-wave analogue of a coherent beamsplitter[18] (see Methods).

The system is allowed to evolve in the double well for a variable time t, before the gases are released by switching off the trapping potential. They expand and interfere after a time-of-flight of 15.7 ms. The resulting interference pattern allows us to extract the relative phase $\phi(z,t)=\theta_1(z,t)-\theta_2(z,t)$ along the length of the system (Fig. 1). Here, $\theta_1(z,t)$ and $\theta_2(z,t)$ are the phase profiles of the two individual gases. Repeating this procedure approximately 150 times for each value of t, we determine the two-point relative phase correlation function

$$C(\bar{z}=z-z',t)=\mathrm{Re}\,\langle e^{i\phi(z,t)-i\phi(z',t)}\rangle$$

It measures the degree of correlation between the phases at two arbitrary points z and z', separated by a distance $\bar{z}$ (refs 22,23). In contrast to the integrated visibility of the interference pattern, which was used in a previous experiment to identify the prethermalized state[4], the phase correlation function provides a sensitive probe for the local dynamics, and is therefore ideally suited to study the propagation of correlations.

Typical experimental data are presented in Fig. 2a. Directly after the quench, the phase correlation function $C(\bar{z},t)$ is close to unity for any distance $\bar{z}$. This is a direct manifestation of the long-range phase coherence produced by the splitting process. After a given evolution time t, the phase correlation function decays exponentially up to a characteristic distance $\bar{z}_c$ and stays nearly constant afterwards: $C(\bar{z}>\bar{z}_c,t)=C(\bar{z}_c,t)$. This means that beyond the distance $\bar{z}_c$ long-range phase coherence is retained across the system. With longer evolution time, the position of $\bar{z}_c$ shifts to larger distances and the value of $C(\bar{z}>\bar{z}_c,t)$ gradually decreases. This evolution continues until the system ...

LETTER
nature
doi:10.1038/nature107

Light-cone-like spreading of correlations in a quantum many-body system

Marc Cheneau[1], Peter Barmettler[2], Dario Poletti[2], Manuel Endres[1], Peter Schauß[1], Takeshi Fukuhara[1], Christian Gross[1], Immanuel Bloch[1,3], Corinna Kollath[2,4] & Stefan Kuhr[1,5]

In relativistic quantum field theory, information propagation is bounded by the speed of light. No such limit exists in the non-relativistic case, although in real physical systems, short-range interactions may be expected to restrict the propagation of information to finite velocities. The question of how fast correlations can spread in quantum many-body systems has been long studied[1]. The existence of a maximal velocity, known as the Lieb-Robinson bound, has been shown theoretically to exist in several interacting many-body systems (for example, spins on a lattice[2–5])—such systems can be regarded as exhibiting an effective light cone that bounds the propagation speed of correlations. The existence of such a 'speed of light' has profound implications for condensed

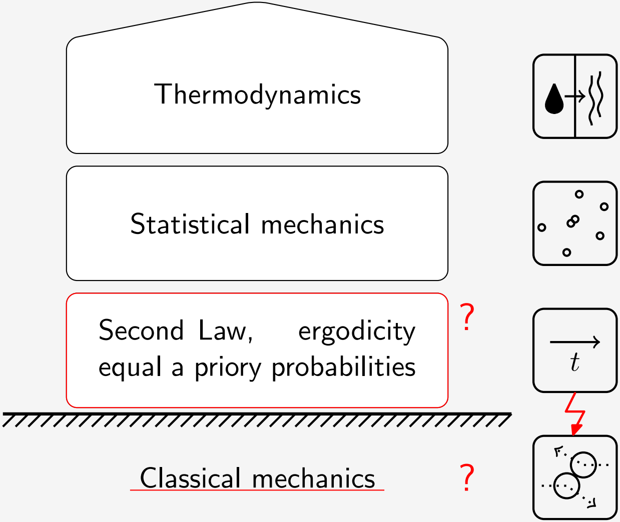
the one-dimensional geometry considered here, the critical point this transition is located at $(U/J)_c \approx 3.4$ (ref. 22). We observed the ti evolution of spatial correlations after a fast decrease of the effecti interaction strength U/J, from an initial value deep in the Mo insulating regime, with filling $n=1$, to a final value closer to the criti point (Fig. 1a). After such a quench, the initial many-body state $|\Psi_0\rangle$ highly excited and acts as a source of quasiparticles. In order elucidate the nature and the dynamics of these quasiparticles, we ha developed an analytical model in which the occupancy of each latt site is restricted to $n=0$, 1 or 2 (Supplementary Information). F large interaction strengths, the quasiparticles consist of either an exce particle ('doublon') or a hole ('holon') on top of the unity-filling bac

nature physics
ARTICLES
PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NPHYS2232

Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas

S. Trotzky[1,2,3]*, Y-A. Chen[1,2,3], A. Flesch[4]*, I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7,8] and I. Bloch[1,2,3]

Motivation III: A new foundation for statistical mechanics



Motivation III: A new foundation for statistical mechanics

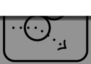
"There is no line of argument proceding from the laws of microscopic mechanics to macroscopic phenomena that is generally regarded by physicists as convincing in all respects."

— E. T. Jaynes [2] (1957)

"Statistical physics [...] has not yet developed a set of generally accepted formal axioms [...]"

— Jos Uffink [3] (2006)

Classical mechanics ?



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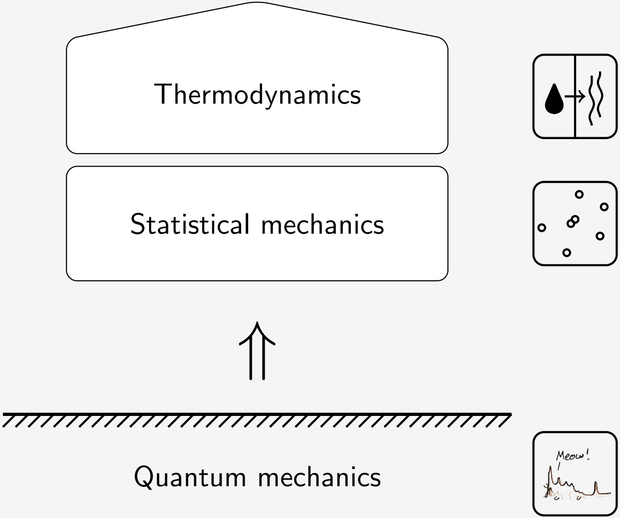


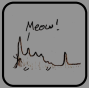
Motivation III: A new foundation for statistical mechanics

Thermodynamics

!!! Reviews !!! Reviews !!! Reviews !!! Reviews !!! Reviews !!!

- Shallow but broad but overview:
  J Eisert, M Friesdorf, and C Gogolin, Nature Physics, 11 (2014), 124–130
- In depth review:
  C. Gogolin and J. Eisert (2015), arXiv: 1503.07538v1

Quantum mechanics



Equilibration & thermalization

Equilibration

Theorem (Equilibration on average [12])

If H has non-degenerate energy gaps, then for every $\rho(0) = |\psi_0\rangle\langle\psi_0|$ there exists a ω^S such that:

$$\overline{\mathcal{D}\left(\rho^S(t), \omega^S\right)} \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\text{eff}}}}$$

[10] M. Cramer, C. M. Dawson, J. Eisert, and T. J. Osborne, Physical Review Letters, 100.3 (2008), 30602
[11] P. Reimann, Physical Review Letters, 101.19 (2008), 190403
[12] N. Linden, S. Popescu, A. Short, and A. Winter, Physical Review E, 79.6 (2009), 61103
[13] A. J. Short and T. C. Farrelly, New Journal of Physics, 14.1 (2012), 013063
[14] P. Reimann and M. Kastner, New Journal of Physics, 14.4 (2012), 043020

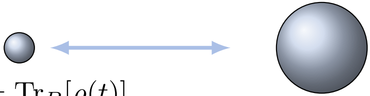
Equilibration

Theorem (Equilibration on average [12])

If H ha ... $\psi_0|$
there e ...

Setting

d_S $\qquad$ $d_B \gg d_S$



$$\rho^S(t) = \mathrm{Tr}_B[\rho(t)]$$

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Effective dimension

$$d^{\mathrm{eff}} = \frac{1}{\sum_k |\langle E_k|\psi_0\rangle|^4}$$

Intuition: Dimension of supporting energy subspace

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If H has non-degenerate energy gaps, then for every $\rho(0) = |\psi_0\rangle\langle\psi_0|$ there exists a ω^S such that:

$$\overline{\mathcal{D}\left(\rho^S(t), \omega^S\right)} \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\mathrm{eff}}}}$$

$\Longrightarrow$ If $d^{\mathrm{eff}} \gg d_S^2$ then $\rho^S(t)$ equilibrates on average.

[10] M. Cramer, C. M. Dawson, J. Eisert, and T. J. Osborne, Physical Review Letters, 100.3 (2008), 30602
[11] P. Reimann, Physical Review Letters, 101.19 (2008), 190403
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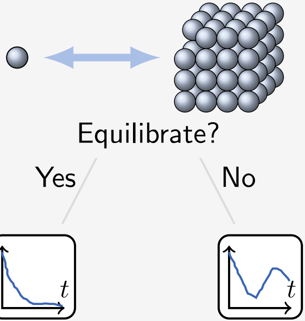
Thermalization



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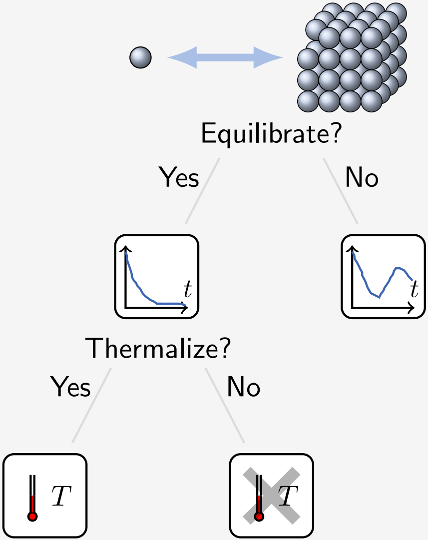

Thermalization is a complicated process

Thermalization implies:

1. Equilibration [10–15]
2. Subsystem initial state independence [16, 17]
3. Weak bath state dependence [18]
4. Diagonal form of the subsystem equilibrium state [19]
5. Thermal state $\omega^S = \mathrm{Tr}_B[\omega] \approx g^S_{H_S}(\beta) \propto \mathrm{e}^{-\beta\, H_S}$ [18, 20, 21]

[10] M. Cramer, C. M. Dawson, J. Eisert, and T. J. Osborne, Physical Review Letters, 100.3 (2008), 30602
[11] P. Reimann, Physical Review Letters, 101.19 (2008), 190403
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[14] P. Reimann and M. Kastner, New Journal of Physics, 14.4 (2012), 043020
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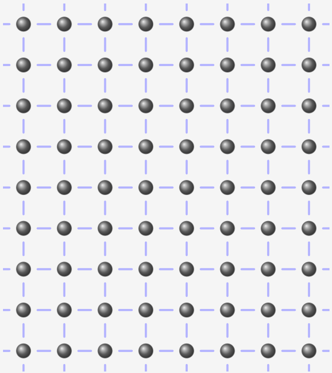
[18] A. Riera, C. Gogolin, and J. Eisert, Physical Review Letters, 108.8 (2012), 080402
[19] C. Gogolin, Physical Review E, 81.5 (2010), 051127
[20] M. P. Mueller, E. Adlam, L. Masanes, and N. Wiebe (2013), arXiv: 1312.7420
[21] F. G. S. L. Brandão and M. Cramer (2015), arXiv: 1502.03263v1
...

Locality of temperature

The setting

- Local Hamiltonian (spins or fermions)

$$H := \sum_{\lambda \in \mathcal{E}} h_\lambda$$



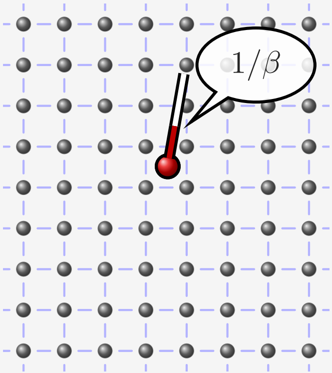
The setting

- Local Hamiltonian (spins or fermions)

$$H := \sum_{\lambda \in \mathcal{E}} h_\lambda$$

- Thermal state

$$g(\beta) := \frac{e^{-\beta H}}{\mathrm{Tr}\left[e^{-\beta H}\right]}$$



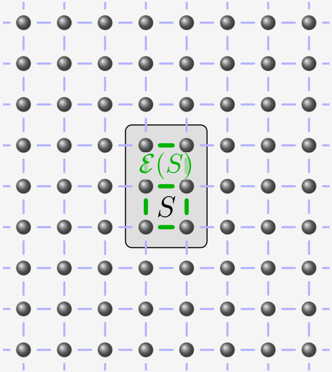
The setting

- Local Hamiltonian truncated to $S \subset V$

$$H_S := \sum_{\lambda \in \mathcal{E}(S)} h_\lambda$$

- Thermal state

$$g\ \ (\beta) := \frac{\mathrm{e}^{-\beta\ H}}{\mathrm{Tr}\left[\mathrm{e}^{-\beta\ H}\ \ \right]}$$



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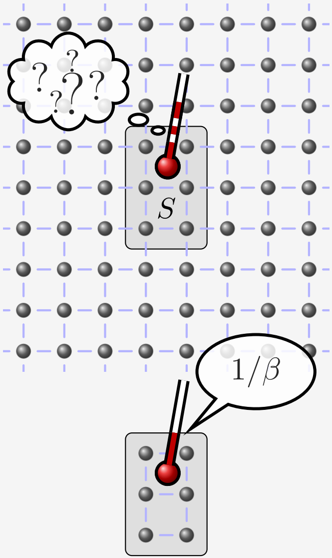


The setting

- Local Hamiltonian truncated to $S \subset V$

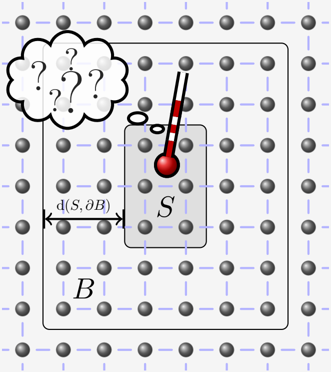
$$H_S := \sum_{\lambda \in \mathcal{E}(S)} h_\lambda$$

- Thermal state

$$g_B(\beta) := \frac{\mathrm{e}^{-\beta\, H_B}}{\mathrm{Tr}\left[\mathrm{e}^{-\beta\, H_B}\right]}$$

- Introduce buffer region

$$\mathrm{Tr}_{S^c}[g_B(\beta)] \approx \mathrm{Tr}_{S^c}[g(\beta)]\ ?$$



This can be made rigorous:

Generalized covariance

$$\operatorname{cov}_\rho^\tau(A, A') := \operatorname{Tr}[\rho^\tau A \, \rho^{1-\tau} A'] - \operatorname{Tr}[\rho \, A] \operatorname{Tr}[\rho \, A']$$

[22] M. Kliesch, C. Gogolin, M. J. Kastoryano, A. Riera, and J. Eisert (2013), arXiv: 1309.0816v2

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Theorem (Truncation formula [22])

For any observable $A = A_S \otimes \mathbb{1}$

$$\operatorname{Tr}[A\, g_B(\beta)] - \operatorname{Tr}[A\, g(\beta)] = \beta \int_0^1 \int_0^1 \operatorname{cov}^\tau_{g(s,\beta)}(H_{\partial B}, A)\, \mathrm{d}\tau\, \mathrm{d}s\,,$$

where $g(s, \beta)$ is thermal state of $H(s) := H - (1 - s)\ H_{\partial B}$.

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$$\operatorname{cov}^{\tau}_{\rho}(A, A') := \operatorname{Tr}[\rho^{\tau} A\, \rho^{1-\tau} A'] - \operatorname{Tr}[\rho\, A]\, \operatorname{Tr}[\rho\, A']$$

exactly captures the response of local expectation values.

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Clustering of correlations

Theorem (Clustering of correlations at high temperature [22])

Let $J := \max_\lambda \|h_\lambda\|_\infty$, then for every $\tau \in [0,1]$ and $\beta < \beta^*(J,\alpha)$

$$\left|\mathrm{cov}^{\tau}_{g(\beta)}(A, A')\right| \leq C\ \mathrm{e}^{-\,\mathrm{d}(A,A')\,/\,\xi(\beta\,J,\alpha)}$$

with $\alpha = \alpha(\mathcal{E})$ the lattice animal constant.

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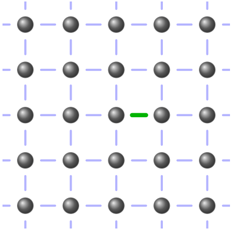

$$\#\text{animals}(m) \leq \alpha^m$$

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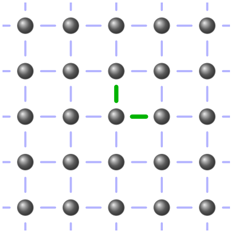

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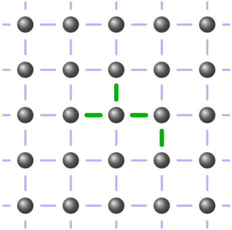

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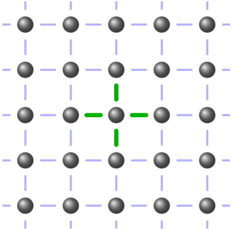

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with $\alpha = \alpha(\mathcal{E})$ the lattice animal constant.

$$\boxed{\implies \mathcal{D}\left(g^S(\beta), g_B{}^S(\beta)\right) \leq C'\ \mathrm{e}^{-\mathrm{d}(S,\partial B)/\xi(\beta\,J,\alpha)}}$$

[22] M. Kliesch, C. Gogolin, M. J. Kastoryano, A. Riera, and J. Eisert (2013), arXiv: 1309.0816v2

Implications

$$\boxed{\beta < \beta^*(J,\alpha) \implies \mathcal{D}\left(g^S(\beta), {g_B}^S(\beta)\right) \leq C' \, \mathrm{e}^{-\mathrm{d}(S,\partial B)/\xi(\beta\, J,\alpha)}}$$

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$g^S(\beta)$ only depends exponentially weakly on far away terms of the Hamiltonian.

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Classical simulability with cost independent of total system size

Local expectation values can be calculates with cost independent of the total system size.

A universal bound on phase transitions

Universal critical temperature

The critical temperature

$$\frac{1}{\beta^* J} = \frac{2}{\ln\left((1+\sqrt{1+4/\alpha})/2\right)}$$

upper bounds the physical critical temperatures of all possible models.

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Example: 2D square lattice ($\alpha \leq 4\,\mathrm{e}$)

- The bound:
  $1/(\beta^*\,J) = 2/\ln((1+\sqrt{1+1/\,\mathrm{e}})/2) \approx 24.58$
- Ising model (ferromagnetic, isotropic) phase transition at:
  $1/(\beta_c\,J) = 2/\ln(1+\sqrt{2}) \approx 2.27$

Conclusions

Conclusions

- Ongoing program reconsidering the foundations of statistical mechanics
- Well connected to exciting experiments
- Rigorous results on equilibration, thermalization, locality of temperature, . . .

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Thank you for your attention!

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