Boson-Sampling in the light of sample complexity: a review

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Why BosonSampling?

As building a quantum computer is difficult, can we demonstrate quantum supremacy by simpler means?

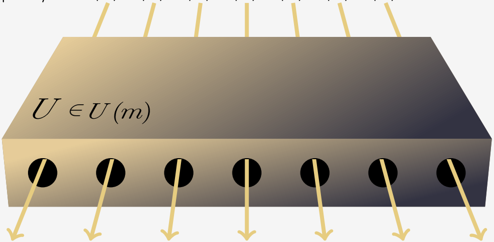
Physical BosonSampling

m modes

n bosons

Input: Fock state $|\mathbb{1}_n\rangle = |1\rangle\ |1\rangle\ |1\rangle\ |0\rangle\ |0\rangle\ |0\rangle\ |0\rangle$

U and n define an instance of BosonSampling.



Output: sample $S = (\ s_1\ ,\ s_2\ ,\ s_3\ ,\ s_4\ ,\ s_5\ ,\ s_6\ ,\ s_7\ )$

The abstract sampling problem

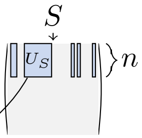
BosonSampling

Given n and a fixed $m \times m$ unitary matrix U, sample from

$$\{S = (s_1, \ldots, s_m) \colon s_j \geq 0 \wedge \sum_j s_j = n\}$$

according to

$$\Pr_{\mathcal{D}_U}[S] := |\,\mathrm{Perm}(U_S)|^2 / \prod_{j=1}^{m} (s_j!). \qquad U =$$



Approximate BosonSampling is hard

Ref. [2] provides strong evidence that:

Hardness of BosonSampling

If 1-norm approximate classical efficient BosonSampling were possible for Haar random U and $m \in \Omega(n^5)$ then the polynomial hierarchy would collapse to the third level.

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[2] S. Aaronson and A. Arkhipov, in: Proceedings of the 43rd annual ACM symposium on Theory of computing - STOC '11, New York: ACM Press, 2011, p. 333

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- Is almost as unlikely as P=NP

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- Still holds after post selection on bit string outcomes

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Recent attempts of physical realizations. . .

Science 339, 798 (2013):

Boson Sampling on a Photonic Chip

Justin B. Spring,[1]* Benjamin J. Metcalf,[1] Peter C. Humphreys,[1] W. Steven Kolthammer,[1] Xian-Min Jin,[1,2] Marco Barbieri,[1] Animesh Datta,[1] Nicholas Thomas-Peter,[1] Nathan K. Langford,[1,3] Dmytro Kundys,[4] James C. Gates,[4] Brian J. Smith,[1] Peter G. R. Smith,[4] Ian A. Walmsley[1]*

... quantum computers ideally solve problems such as factoring integers ...

LETTERS
PUBLISHED ONLINE: 12 MAY 2013 | DOI: 10.1038/NPHOTON.2013.102
nature photonics

Experimental boson sampling

Max Tillmann[1,2]*, Borivoje Dakić[1], René Heilmann[3], Stefan Nolte[3], Alexander Szameit[3] and Philip Walther[1,2]*

... dramatic increase in ... photons. Randomly chosen instances of this problem are strongly ... classical means. Instances of boson ...

nature photonics
LETTERS
PUBLISHED ONLINE: 26 MAY 2013 | DOI: 10.1038/NPHOTON.2013.112

Integrated multimode interferometers with arbitrary designs for photonic boson sampling

Andrea Crespi[1,2], Roberto Osellame[1,2]*, Roberta Ramponi[1,2], Daniel J. Brod[3], Ernesto F. Galvão[3]*, Nicolò Spagnolo[4], Chiara Vitelli[4,5], Enrico Maiorino[4], Paolo Mataloni[4] and Fabio Sciarrino[4]*

The evolution of bosons undergoing arbitrary ...

Science 339, 794 (2013):

Photonic Boson Sampling in a Tunable Circuit

Matthew A. Broome,[1,2]* Alessandro Fedrizzi,[1,2] Saleh Rahimi-Keshari,[2] Justin Dove,[3] Scott Aaronson,[3] Timothy C. Ralph,[2] Andrew G. White[1,2]

Efficient experimental validation of photonic boson sampling against the uniform distribution

Nicolò Spagnolo,[1] Chiara Vitelli,[1,2] Marco Bentivegna,[1] Daniel J. Brod,[3] Andrea Crespi,[4,5] Fulvio Flamini,[1] Sandro Giacomini,[1] Giorgio Milani,[1] Roberta Ramponi,[4,5] Paolo Mataloni,[1,6] Roberto Osellame,[4,5,*] Ernesto F. Galvão,[3,†] and Fabio Sciarrino[1,6,‡]
[1]Dipartimento di Fisica, Sapienza Università di Roma, Piazzale Aldo Moro 5, I-00185 Roma, Italy
[2]Center of Life NanoScience @ La Sapienza, Istituto Italiano di Tecnologia, Viale Regina Elena, 255, I-00185 Roma, Ita...
[3]Instituto de Físic...
Av. Gal. Milton Tavares ...
[4]Istituto di Fotonica e Nanotecnol...
Piazza Leonardo ...
[5]Dipartimento di Fisica, Politecnico di M...
[6]Istituto Nazionale di Ottica (IN...

A boson sampling device is a specialised ...um computer that solves a problem wh... strongly believed to be computationally ...or classical computers [1]. Recently a ...ber of small-scale implementations have bee... ...orted [2–5], all based on multi-photon int...

On the experimental verification of quantum complexity in linear optics

Jacques Carolan,[1] Pete Shadbolt,[1] Jasmin D. A. Meinecke,[1] Nicholas J. Russell,[1] Nur Ismail,[2] Kerstin Wörhoff,[2] Terry Rudolph,[3] Mark G. Thompson,[1] Jeremy L. O'Brien,[1] Jonathan C. F. Matthews,[1,*] and Anthony Laing[1,†]
[1]Centre for Quantum Photonics, H. H. Wills Physics Laboratory & Department of Electrical and Electronic Engineering, University of Bristol, ... Building, Woodland Road, Bristol, BS8 1UB, UK
...up, MESA+ Institute for Nanotechnology, ... Enschede, The Netherlands
...erial College London, London SW7 2BW, UK

...ember 13, 2013)

...utational problems that are beyond the capabilities ...hat exploit characteristics inherent to a particular ...suited to those characteristics. Evidence exists to ...s, which have propagated through a linear optical ...bility distribution that is intractable to classical ...mplexity of this type of sampling problem means ... task of establishing correct operation becomes one ...l evidence. Here, we develop scalable methods to ...is class of sampling algorithm, which we test with ...hotons, on Hilbert spaces of up to 50k dimensions. ... confident that we are not sampling from a uniform

BosonSampling with Controllable Distinguishability

Max Tillmann[1], Si-Hui Tan[2], Sarah E. Stoeckl[1], Barry C. Sanders[3], Hubert de Guise[4], René Heilmann[5], Stefan Nolte[5], Alexander Szameit[5], Philip Walther[1]
[1]Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria
[2]Singapore University of Technology and Design, 20 Dover Drive, 138682 Singapore
[3]Institute for Quantum Science and Technology, University of Calgary, Alberta, Canada T2N 1N4
[4]Department of Physics, Lakehead University, Thunder Bay, Ontario, P7B 5E1, Canada and
[5]Institute of Applied Physics, Abbe Center of Photonics, ... D-07743 Jena, Germany

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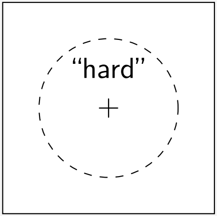
- Calculating $\mathrm{Pr}_{\mathcal{D}_U}[S]$ for fixed S from U is in general hard.
- ⇒ There is no obvious way to certify an implementation.

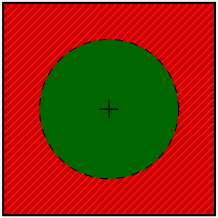
> Particularly high standards should be applied as BosonSampling has no other use!

What is certification?



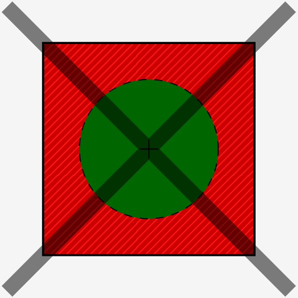
What is certification?

naive



What is certification?

impossible



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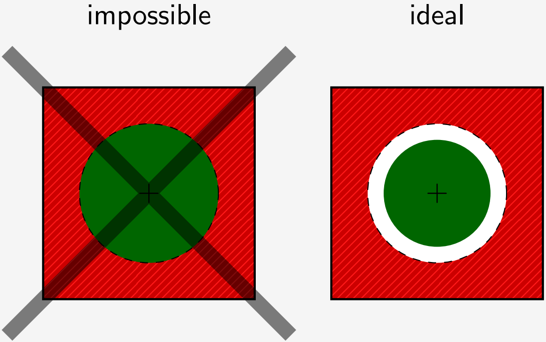

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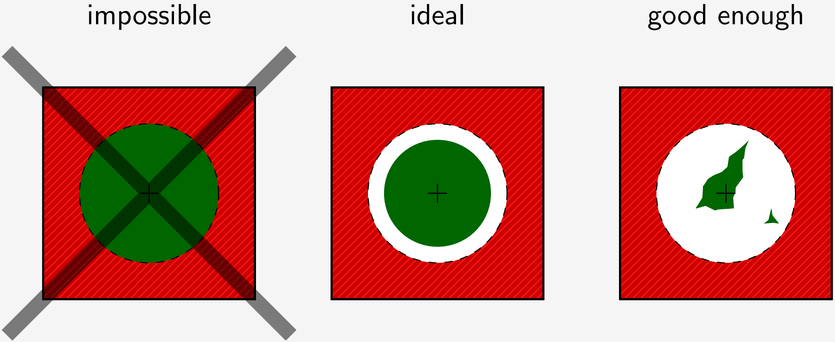

First step: distinguishing from uniform distribution

Simpler task:

Distinguishing from uniform

- Promise: Device either does BosonSampling or samples from the corresponding uniform distribution.
- Question: Decide which is the case.

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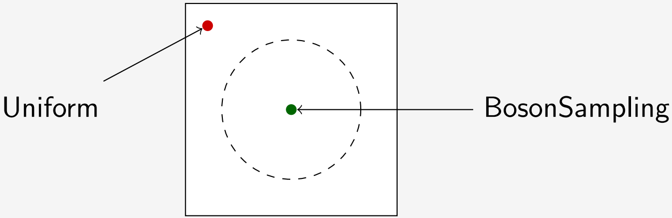

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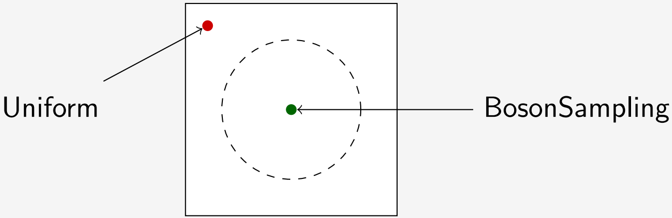

Now look at this in two settings. . .

State discrimination setting

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⇒ Let $m \in \mathrm{O}(\mathrm{poly}(n))$, then any instance of BosonSampling that could be potentially hard to sample 1-norm approximately classically can be distinguished from the uniform distribution from $\mathrm{O}(n^{2+\epsilon})$ samples.

Black box setting

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⇒ If U Haar random and $m \in \Omega(n^{\nu})$ with $\nu > 3$ ($\nu > 2$ with post selection), then with probability supra-exponentially close to one in n no symmetric probabilistic algorithm can distinguish $\mathcal{D}_U$ from the uniform distribution from fewer than $\Omega(\mathrm{e}^{n/2})$ many samples.

Corroboration/partial certification

One can look at:

1. Boson bunching [6, 9]
2. Moments
3. Row norm estimators [1]

...

⇛ Allows to efficiently distinguish from uniform distribution.

⇛ This yields no certification. All algorithms can be fooled!

[1] S. Aaronson and A. Arkhipov, ECCC, 135.135 (2013), TR13–135, arXiv: 1309.7460
[6] C. Gogolin, M. Kliesch, L. Aolita, and J. Eisert (2013), arXiv: 1306.3995
[9] M. C. Tichy, M. Tiersch, F. Mintert, and A. Buchleitner, New Journal of Physics, 14.9 (2012), 093015

This is no coincidence

- ! $\mathcal{D}_U$ typically has a high min-entropy [6].

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- ! $\mathcal{D}_U$ typically has a high min-entropy [6].
- ⇛ Observation of F. Brandao (published in [1] based on [11]):

Existence of an efficiently samplable cheating distribution

For every instance of BosonSampling with high min-entropy and every circuit length $T \in \mathrm{O}(\mathrm{poly}(n))$ there is a classically efficiently samplable distribution indistinguishable from $\mathcal{D}_U$ by all circuits of length T.

[1] S. Aaronson and A. Arkhipov, ECCC, 135.135 (2013), TR13–135, arXiv: 1309.7460
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- Classical certification of BosonSampling is likely difficult.
- BosonSampling is both close and far from uniform.

References

Thank you for your attention!

⟶ slides: www.cgogolin.de

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