Absence of thermalization in non-integrable systems

Christian Gogolin, Arnau Riera, Markus Müller, and Jens Eisert

Dahlem Center for Complex Quantum Systems, Freie Universität Berlin

Workshop “Many-Body Quantum Dynamics in Closed Systems”
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Old questions and new contributions

How do quantum mechanics and statistical mechanics go together?
Many-Body Quantum Dynamics in Closed Systems

Absence of thermalization

\[ \psi_t = e^{-iHt} \psi_0 \]

\[ A_t = \text{Tr}[A|\psi_t\rangle\langle\psi_t|] \]

\[ \psi_{\text{S}} = \text{Tr}[B|\psi_t\rangle\langle\psi_t|] \]

\[ H_{\text{SB}} + H_S \otimes 1 + 1 \otimes H_B \]

Equilibration:
- strong: equilibrated between \( t_1 \) and \( t_2 \) \[1\]
- weak: equilibrated for most times \[2\]

Thermalization:

\[ \psi_{\text{S}} \approx \rho_{\text{Gibbs}} \propto e^{-\beta H_S} \]

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Setup and terminology

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  [Diagram showing equilibration with a graph indicating a transition over time]

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  [Diagram showing thermalization with a graph indicating temperature change]

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Equilibration and a maximum entropy principle
Theorem 1 (Maximum entropy principle [3])

If $\text{Tr}[A \psi_t]$ equilibrates, it equilibrates towards its time average

$$\overline{\text{Tr}[A \psi_t]} = \text{Tr}[A \overline{\psi_t}] = \text{Tr}[A \omega],$$

where $\omega = \sum_k \pi_k \psi_0 \pi_k$

(with $\pi_k$ the energy eigen projectors) is the dephased state that maximizes the von Neumann entropy, given all conserved quantities.

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\[ \Rightarrow \]

Maximum entropy principle from pure quantum dynamics.
Has nothing to do with (non)-integrability.

Time averaging

\[ \psi_0 = \]

Interesting open questions:
Do we really need all (exponentially many) conserved
quantities?
If not, then which?
Does this depend on integrability of the model?
What is the relation to the GGE?

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$\Rightarrow$ Maximum entropy principle from pure quantum dynamics. Has nothing to do with (non)-integrability.

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$\psi_0 = \omega$ is a pinching $\Rightarrow \omega$ maximizes entropy.

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(With $\pi_k$ the energy eigen projectors, it maximizes entropy.)

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Thermalization and integrability
Thermalization is a complicated process

Thermalization implies:

1. Equilibration [2, 4, 5]
2. Subsystem initial state independence [3]
3. Weak bath state dependence [6]
4. Diagonal form of the subsystem equilibrium state [7]
5. Gibbs state $e^{-\beta H}$ [5, 6]

There is a common belief in the literature [8, 9, 10, 11, 12] …

\[
\begin{align*}
\text{Non-integrable} & \implies \text{Thermalization} \\
\text{Integrable} & \implies \text{No thermalization}
\end{align*}
\]

---

Thermalization and quantum integrability

There is a common belief in the literature [8, 9, 10, 11, 12] ... 

Non-integrable $\implies$ Thermalization  
Integrable $\implies$ No thermalization

... but there are problems.

---

Notions of (non-)integrability

A system is with $n$ degrees of freedom is integrable if:

- There exist $n$ (local) conserved mutually commuting linearly independent operators.
- There exist $n$ (local) conserved mutually commuting algebraically independent operators.
- The system is integrable by the Bethe ansatz.
- The system exhibits nondiffractive scattering.
- The quantum many-body system is exactly solvable in any way.
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Lack of imagination?
Reminder on integrability in classical mechanics

**Classical Liouville integrability**

A system with $n$ degrees of freedom is called **integrable** if it entails a maximal set of $n$ independent Poisson commuting constants of motion and is called **non-integrable** otherwise [13].

Reminder on integrability in classical mechanics

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Classical:

\( \text{integrability} \implies \text{systematic solvable}\)
and evolution on a \( n \)-torus

Quantum:

\( \text{always systematic solvable}\)
and evolution on a \( d \)-torus

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**Classical:**
- integrability \( \Rightarrow \) systematic solvable and evolution on a \( n \)-torus
- qualitative question

**Quantum:**
- always systematic solvable and evolution on a \( d \)-torus
- quantitative question?

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Classical:
- integrability $\Rightarrow$ systematic solvable and evolution on a $n$-torus
- qualitative question
- thermalization $\Rightarrow$ non-integrability

Quantum:
- always systematic solvable and evolution on a $d$-torus
- quantitative question?
- thermalization $\Leftrightarrow$ non-integrability

Absence of thermalization in non integrable systems

Result (Theorem 1 and 2 in [3]):

- Too little (geometric) entanglement in the energy eigenbasis prevents initial state independence.
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The model:

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H = \sum_{i=1}^{n} h_i \sigma_i^Z + \sum_{i=1}^{n-1} \vec{b}_i \cdot \vec{\sigma}_{NN}
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Interesting open questions:

- What is the relation to Anderson localization?
- Can this also happen in translation invariant systems?

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\[ |\psi_1\rangle, |\psi_2\rangle \]

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Spin-1/2 XYZ chain with random coupling and on-site field.

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Proving thermalization
Two ways to prove thermalization

\[ \langle E_k | \psi_0 | E_k \rangle \]

Assumptions about:

ETH

Our result

\[ |E_k\rangle \]
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Structure of the argument

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Classical level counting à la Goldstein [14] with no interaction

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\[ \| H_{SB} \|_\infty \ll k_B T \]

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→ Typicality arguments

→ Kinematic

→ Equilibration results

→ Dynamic

Absence of thermalization

The result

\[ \| \mathcal{H}_{SB} \|_\infty \gg \text{gaps}(\mathcal{H}_0) \]
\[ \| \mathcal{H}_{SB} \|_\infty \ll k_B T \ll \Delta \]

\[ \langle E_k | \psi_0 | E_k \rangle \]
\[ \Omega^B_\Delta(E) \]

\[ \Rightarrow \text{“Theorem” 2 (Theorem 2 in [6])} \]

(Kinematic) Almost all pure states from a microcanonical subspace \([E, E + \Delta]\) are locally close to a Gibbs state.

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- Can we capture the intuition behind **non-integrability** in a mathematically **precise definition**?
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- How is this related to the GGE and ETH?
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- How are non-integrability and thermalization related?
Collaborators

Arnau Riera  Martin Kliesch  Jens Eisert

Markus P. Müller
Thank you for your attention!

→ slides: www.cgogolin.de


