Under what conditions do quantum systems thermalize?

New insights from quantum information theory

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Irreversibility from quantum many body dynamics
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Under what conditions do quantum systems thermalize?

Irreversibility from unitary dynamics

Equilibrate?

Yes

No
Irreversibility from quantum many body dynamics

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Thermalize?

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\( T \)

\( \overline{T} \)
Irreversibility from quantum many body dynamics

Under what conditions do quantum systems thermalize?

Yes

Quantum computer

No

Thermalize?

Yes

Equilibrate?

No

$t$

$T$

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$t$
New foundation for statistical mechanics

- Thermodynamics
- Statistical Mechanics
- Second Law ergodicity equal a priory probabilities
- Classical Mechanics
New foundation for statistical mechanics

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Thermodynamics

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Quantum Mechanics
Thermalization is a complicated process

Thermalization implies:
1. **Equilibration** [1, 2, 3]
2. **Subsystem initial state independence** [4]
3. **Weak bath state dependence** [5]
4. **Diagonal form of the subsystem equilibrium state** [6]
5. **Gibbs state** \( e^{-\beta \mathcal{H}} \) [3, 5]

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Under what conditions do quantum systems thermalize?

Thermalization and quantum integrability

There is a common belief in the literature [7, 8, 9, 10, 11] …

| Non-integrable | \( \implies \) | Thermalization |
|----------------|-----------------|
| Integrable     | \( \implies \)  | No thermalization |

There is a common belief in the literature \cite{7, 8, 9, 10, 11} ... which is unfortunately not quite true.

\begin{center}
\begin{tabular}{cc}
Non-integrable & $\Rightarrow$ & Thermalization \\
Integrable & $\Rightarrow$ & No thermalization
\end{tabular}
\end{center}

\footnotesize
\begin{itemize}
    \item \cite{7} C. Kollath et. al PRL 98, (2007) 180601
    \item \cite{8} S. Manmana, S. Wessel, R. Noack, and A. Muramatsu, ibid. 98 (2007) 210405
    \item \cite{9} M. Rigol, V. Dunjko, and M. Olshanii, Nature 452 (2008) 854
    \item \cite{10} M. C. Banuls, J. I. Cirac, and M. B. Hastings, arXiv:1007.3957
    \item \cite{11} M. Rigol, PRL 103, (2009) 100403
\end{itemize}
Absence of thermalization in non integrable systems

Result (Theorem 1 and 2 in [4]):

- Too little (geometric) entanglement in the energy eigenbasis prevents initial state independence.
- This can happen even in non-integrable systems.

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\[ D(S, B) \geq D(S, B) - R(|\psi_1\rangle) - R(|\psi_2\rangle). \]

Corollary of Theorem 2 in [4]:

Let \( \{ |i\rangle \} \) be a basis for \( S \), then if \( \delta = \max_k \min_i D(\text{Tr} B |E_k\rangle \langle E_k|, |i\rangle \langle i|) \) is small, then for all \( |i\rangle \) and almost all \( |\psi_B\rangle \)

\[ R(|i\rangle \otimes |\psi_B\rangle) \leq 4 \delta. \]

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Effective entanglement in the eigenbasis (for spin 1/2)

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\[
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D(\uparrow, \downarrow) \geq D(\uparrow, \downarrow) - R(|\psi_1\rangle) - R(|\psi_2\rangle).
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- **This can happen even in non-integrable systems.**

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\mathcal{D}(\uparrow, \downarrow) \geq \mathcal{D}(\uparrow, \downarrow) - R(|\psi_1\rangle) - R(|\psi_2\rangle).
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Conclusions and outlook

We have seen in this talk:

Non-integrability \( \Rightarrow \) Thermalization

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But there is more:

- Rigorous results on Equilibration [1, 2]
- A strong connection to decoherence [6]
- A quantum algorithm to prepare Gibbs states [5].
- Thermalization in exactly solvable models [12, 13]

Conclusions and outlook

We have seen in this talk:

Non-integrability $\not\Rightarrow$ Thermalization

But there is more:

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The major open question:

- Time scales. How long does it take to equilibrate?

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Collaborators

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Arnau Riera
Jens Eisert
Peter Janotta
Haye Hinrichsen
Andreas Winter
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References

Thank you for your attention!

slides: www.cgogolin.de